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A ROBUST CONTROL OF A DYNAMIC BEAM STRUCTURE WITH TIME DELAY EFFECT

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The objective of this paper is to present a feasible methodology that can achieve good control performance of a dynamic beam structure system with time delay effect. To achieve good control, a robust control called the modified fuzzy sliding mode control (FSMC) with phase shift method is introduced in this paper. Conventionally, the FSMC is easily designed without precise system modelling and needs very little information for online calculation. However, numerical simulation results show that increasing the time delay will reduce its performance and the system will become unstable for a large time delay. The phase shift method can assist the system in predicting the real status value as a compensation for time delay effect. The results also show improvement on both the system's robustness and stability. Furthermore, by comparing the maximum and average bending displacement reduction rates of the beam structure, it is found that the FSMC with phase shift compensation will improve the system's performance.

1. INTRODUCTION

Beams are widely used in machines, architectural structures and aircrafts. The dynamic behavior becomes more complicated due to edging operating environments and more severe dynamic loads, for instance, high-speed aircrafts undergoing high attack angle or turbulences. Therefore, proper active control is essential in order to ensure that the beam structure stays will constrained. Besides, time delay always exists in feedback control loop due to sensing or computation process. The problem of the system's dynamic stability, which is caused by the time delay effect, has made the system more controllable and unstable. Thus, the issue of how to enhance the system's robustness to the time delay effect is worth further study. In order to obtain a proper control force exerted on the system, accurate and optimal computation is necessary. However, it will cause time delay in the system in the actuating force. The results will not match the system's need on real time. Therefore, the need for a methodology that is simple in calculation and effective in control is critical.

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A simple review of the structure control is stated as follows. Yao [1] introduced the concept of active beam structure control. Yang [2] studied the optimal control for structures under random dynamic loads. Abdel-Roham and Leipholz [3, 4] presented a control law for active structure control, which is based on the optimal control theory. An optimal closed-loop control law was designed for system tracking and regulating. However, the optimal law will bring further time delay due to the time requirement of a large amount of online calculation. Therefore, a methodology that can deal with time delay becomes the key to the practical application of dynamic beam structure control.

In recent years, stabilization of systems with time delay has received considerable attention. Several linear state feedback controllers have been proposed by Su [5], Chou [6] and Shen [7]. The fundamental designs are based on (1) pole placement approach, (2) Lyapunov approach, and (3) linear quadratic regulator approach. In these cases, time delay can be the source of instability. Basharkhah and Yao [8] found that time delay could make the control system lose its reliability. In 1985, Abdel-Roham [9] applied the pole placement method to compensate for the system's time delay. Lo'pez-Almansa and Rodellar [10] applied independent mode space control for predictive control, and their experiments showed that when the number of sensors and actuators is less than the system's mode number, the control system becomes unstable if the time delay effect is taken into account.

The dynamic behavior of a beam structure is represented by a fourth order partial differential equation, which can be reduced to one ordinary differential equation via the assumed mode method for simplification purpose. However, it is noted that when a distributed parameter system is approximated by a lumped parameter system, it will induce further system uncertainties. Then the typical control law is no longer suitable for this situation. In this paper, robust sliding mode control is proven to be an outstanding control law to overcome the problem.

Hwang and Lin [11] presented a fuzzy controller designed with fuzzy sliding surface. This method enjoys the advantages of simple calculation without delicate system modelling. In this research, a fuzzy control is adopted so that chattering induced in sliding mode control can be avoided. The integration of fuzzy algorithm with sliding mode control will secure system stability by sliding mode control and will save a lot of calculation by fuzzy look-up tables. Choi and Kim [12] offered a discrete time fuzzy sliding mode control method for smart structures, and showed that the system can achieve its robustness and that chattering was attenuated.

In this work, we focus on the application of the FSMC with phase shift method to deal with a dynamic beam structure system with time delay effect. First, a control state-space equation of a dynamic beam model is derived in section 2. Then, the FSMC is employed to control the structure and phase shift compensation is used to compensate phase lag due to time delay. Finally, simulation results are presented to compare with the results by passive control and the FSMC without phase shift.

2. DYNAMIC MODEL OF A BEAM STRUCTURE

A dynamic beam structure under active control by a servomechanism is shown in Figure 1. The dynamic load P moves along the longitudinal direction of the beam with a velocity V. The dynamic equation is shown as follows:

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} + C\frac{\partial y}{\partial t} = P\delta(x - Vt) + M_o\delta'(x - a) - M_o\delta'(x - L + a). \quad (1)$$

In the equation, δ is the Dirac delta function and δ' is the derivative. Control torque M_o is exerted by the servomechanism, installed beneath the central portion of the beam at

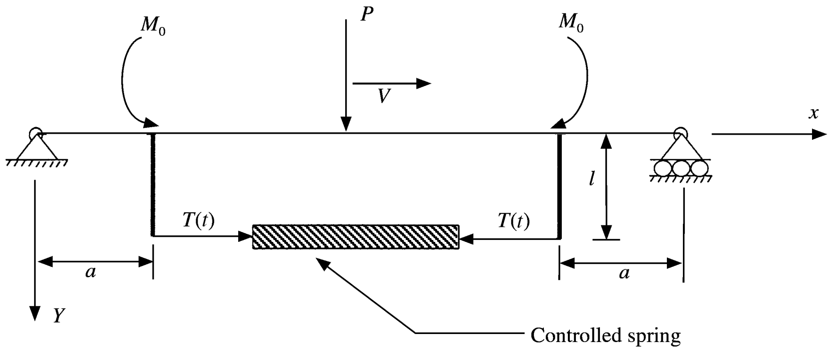

Figure 1. Dynamic beam structure.

a distance a measured from both end supports, and tends to balance the bending phenomenon caused by the moving load.

When the servomechanism is in action, the actuator will increase or decrease the spring displacement according to the control system's needs. The terms $M_o\delta'(x-a) - M_o\delta'(x-L+a)$ are the control forces produced by the servomechanism.

This active control torque M_o is designated as

$$M_o = lS\Delta(t) = Sl\left[u(t) + l\,\frac{\partial y(a,t)}{\partial x} - l\,\frac{\partial y(L-a,t)}{\partial x}\right], \tag{2}$$

where S is the stiffness of the spring, $u(t)$ is the spring displacement caused by servomechanism and $\Delta(t)$ is the displacement of the spring.

If $u(t) = 0$ of equation (2), it is a passive structure control system. Using the assumed mode method, equation (1) for the jth mode can be obtained as

$$\ddot{Y}_j(t) + 2\zeta\omega_j\dot{Y}_j(t) + \omega_j^2 Y_j(t) = \begin{cases} \dfrac{2P}{mL}\sin j\omega t, & j = 2,4,6\ldots, \\ \dfrac{2P}{mL}\sin j\omega t - \dfrac{4j\pi M_o(t)}{mL^2}\cos\dfrac{j\pi a}{L}, & j = 1,3,5\ldots. \end{cases}$$

It can be rewritten as

$$\ddot{Y}_j(t) + 2\zeta\omega_j\dot{Y}_j(t) + \omega_j^2 Y_j(t) = \frac{2P}{mL}\sin(j\omega t) - \frac{4j\pi M_o(t)}{mL^2}\cos\left(\frac{j\pi a}{L}\right)\sin\left(\frac{j\pi}{2}\right), \tag{3}$$

where $Y_j(t) = (2/L)\int_0^L y(x,t)\sin(j\pi x/L)\,\mathrm{d}x$ is the beam displacement of the jth mode, ζ is the damping ratio, $\omega_j = ((j^4\pi^4/L^4)\,(EI/m))^{1/2}$ is the natural angular frequency of the jth mode, $\omega = \pi V/L$ is the natural angular frequency, and

$$M_o(t) = Sl\left[u(t) + 2l\sum_{j=1,3,5\ldots}^{\infty}\frac{j\pi}{L}\cos\frac{j\pi a}{L}\,Y_j(t)\right]. \tag{4}$$

Since the high order modes of motion contribute little to bending displacements, only the basic and the second modes are considered here. Then the equation of motion in the state-space form can be written as

$$\dot{\mathbf{X}}_j = \mathbf{A}_j\mathbf{X}_j + \mathbf{B}_j\mathbf{U}_j + \mathbf{D}_j\mathbf{E}_j, \quad j = 1, 2, \tag{5}$$

where

$$\mathbf{X}_j = \begin{bmatrix} x_{1j} \\ x_{2j} \end{bmatrix} = -\begin{bmatrix} Y_j(t) \\ \dot{Y}_j(t) \end{bmatrix}, \quad j = 1, 2,$$

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 - b_1c_1 & -2\zeta\omega_1 \end{bmatrix}, \qquad b_1 = \frac{4Sl}{mL}\frac{\pi}{L}\cos\frac{\pi a}{L}, \qquad c_1 = \frac{2l\pi}{L}\cos\frac{\pi a}{L},$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ -\omega_2^2 - b_2c_2 & -2\zeta\omega_2 \end{bmatrix}, \qquad b_2 = \frac{8Sl}{mL}\frac{\pi}{L}\cos\frac{2\pi a}{L}\sin\pi, \qquad c_2 = 0,$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 \\ b_1 \end{bmatrix}, \qquad \mathbf{B}_2 = \begin{bmatrix} 0 \\ b_2 \end{bmatrix},$$

$$\mathbf{D}_1 = \mathbf{D}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \qquad U_1 = U_2 = u(t) \quad \text{and} \quad E_j = \frac{2P}{mL}\sin(j\omega t), \; j = 1, 2.$$

A continuous time multi-input–multi-output (MIMO) linear time-invariant system is described by the following equation:

$$\begin{aligned} \dot{\mathbf{X}} &= \mathbf{AX} + \mathbf{BU} + \mathbf{DE}, \\ \mathbf{Y} &= \mathbf{CX}, \end{aligned} \tag{6}$$

where

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{A}_2 \end{bmatrix},$$

in which

$$\mathbf{O} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ b_1 & 0 \\ 0 & 0 \\ 0 & b_2 \end{bmatrix}, \qquad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\mathbf{U} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t), \qquad \mathbf{E} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \quad \text{and} \quad \mathbf{Y} = \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{bmatrix}.$$

The state-space equation as stated in equation (6) will be the dynamic model for control simulation in this research.

3. CONTROL METHODOLOGY

Based on the state-space equation (5) of the system model, the objective of control force U is to control the output states of X to be zero. In the control methodology, the first step is to apply the FSMC. Then, the phase shift method is applied to rotate the state-space phase co-ordinate to compensate for the phase lag due to the system's time delay.

The overall aim of the sliding mode control is to drive the system state from an initial condition $X(0)$ to the state-space origin as $t \to \infty$. The jth component $U_j (j = 1, 2)$ of the state feedback control vector $U(t)$ is discontinuous at the jth switching surface, which is the hyperplane M_j passing through the state origin. The hyperplanes are defined as

$$M_j = \{X_j : C_j X_j = 0\}, \quad (j = 1, 2),$$

where X_j is a vector of states and C_j is a sliding coefficient with row vector. The sliding mode occurs when the state lies in M_j for all j, i.e., in the sliding subspace.

$$M = \bigcap_{j=1}^{2} M_j.$$

Therefore, the sliding function σ_j is defined by the following equation:

$$\sigma_j(t) = c_{1j} e_{1j}(t) + c_{2j} e_{2j}(t) = \mathbf{C}_j \mathbf{e}_j(\mathbf{t}) = \mathbf{C}_j \mathbf{X}_j, \quad j = 1, 2, \tag{7}$$

where $C_j = [c_{1j} \ c_{2j}]$ is the sliding coefficient vector,

$$\mathbf{e}_j(\mathbf{t}) = \begin{bmatrix} e_{1j}(t) \\ e_{2j}(t) \end{bmatrix} = \begin{bmatrix} x_{1j} - 0 \\ x_{2j} - 0 \end{bmatrix} = \mathbf{X}_j - \mathbf{O}$$

is the error vector function, represented by the difference between the state x_j and the zero state $\mathbf{O}$ of control aim for the jth mode. The rate of sliding function $\dot{\sigma}_j$ is the derivative of σ_j.

If $\sigma_j = 0$ in equation (7), it represents the hyperplane. For the sliding mode control, a hitting process is taken to hit the system trajectory into the sliding plane. Then, based on the Routh–Hurwitz criterion for system stability, $\sigma_j \, \dot{\sigma}_j < 0$, the system trajectory will slide along the plane and gradually converges to the origin. It is convenient to take $\sigma_j = S_1$, and $\dot{\sigma}_j = S_2$. Due to the characteristics of the sliding mode control, system chattering will always occur around the balance point. Here a fuzzy set is introduced to minimize this system chattering phenomenon. Based on the criteria of sliding mode [11], it divides the sliding variables S_1 and S_2 and control force $u(t)$ into seven fuzzy variables as (1) [PB]: positive big, (2) [PM]: positive medium, (3) [PS]: positive small, (4) [ZE]: zero, (5) [NB]: negative big, (6) [NM]: negative medium, (7) [NS]: negative small (see Figure 2). For simplicity, a triangular-type assignment function is chosen to assign each variable and these variables are centrally weighted and span over [-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6] in 13 stages.

The linguistic control rules of the center of gravity method are used to form a look-up table as shown in Table 1 [11].

Furthermore, in order to compensate for time delay the system's state-space phase co-ordinate is shifted corresponding to delay time. This shifted phase co-ordinate is used to predict the system's real state value. Then, the FSMC controller can calculate the control force as needed in real time.

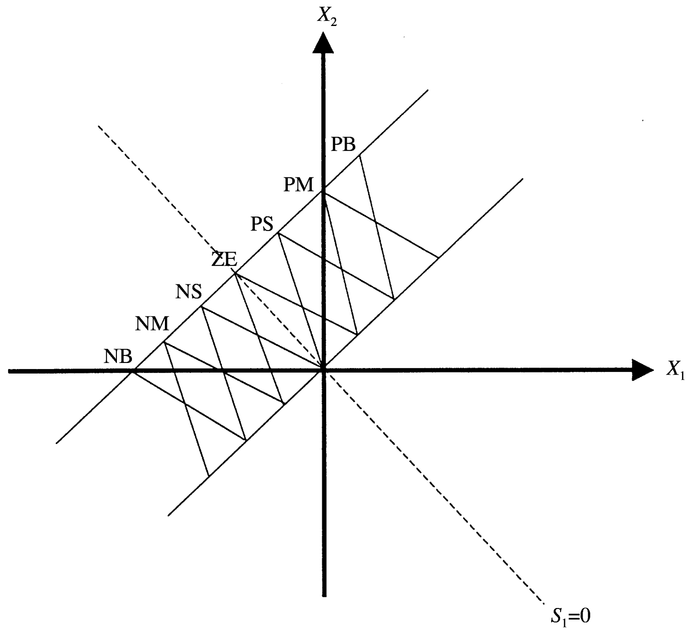

Figure 2. Representation of FSMC.

TABLE 1

7 × 7 fuzzy response table

Look-up table		S_1						
		NB	NM	NS	ZE	PS	PM	PB
	NB	−1	−0·7	−0·5	−0·1	−0·1	−0·1	0
	NM	−0·7	−0·5	−0·5	−0·1	−0·1	0	0·1
	NS	−0·5	−0·5	−0·5	−0·1	0	0·1	0·1
S_2	ZE	−0·1	−0·1	−0·1	0	0·1	0·1	0·1
	PS	−0·1	−0·1	0	0·1	0·5	0·5	0·5
	PM	−0·1	0	0·1	0·1	0·5	0·5	0·7
	PB	0	0·1	0·1	0·1	0·5	0·7	1

In this case, delay time t_d will produce phase lag $w_1 t_d$. However, control force is calculated according to the observed feedback before time delay. Without correct calculation to match the system's needs in real time, the control force will be improper and result in poor control. In order to design a proper control force, the real time state value $X(t)$ has to be predicted. Here, the system's phase co-ordinate is rotated counter-clockwise $w_1 t_d$ degrees of the system phase angle in order to compensate for phase lag. This phase shift effect can be represented by a transfer matrix T, and the sliding function σ_j for the jth mode can be modified as follows:

$$\dot{\sigma}_j = T_j \sigma_j = T_j C_j X_{jd}, \quad (j = 1, 2). \tag{8}$$

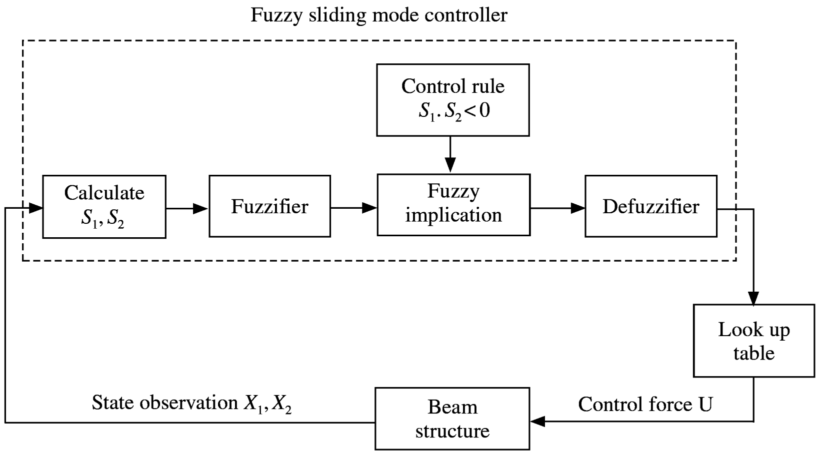

Figure 3. Fuzzy sliding mode control loop.

Here, $X_{jd} = X_j(t - t_d)$ is the state value with time delay and the transfer matrix T_j is

$$T_j = \begin{bmatrix} \cos \omega_j \sqrt{1-\xi^2} t_d + \dfrac{\xi}{\sqrt{1-\xi^2}} \sin \omega_j \sqrt{1-\xi^2} t_d & \dfrac{1}{\omega_j \sqrt{1-\xi^2}} \sin \omega_j \sqrt{1-\xi^2} t_d \\ -\dfrac{\omega_j}{\sqrt{1-\xi^2}} \sin \omega_j \sqrt{1-\xi^2} t_d & \cos \omega_j \sqrt{1-\xi^2} t_d - \dfrac{\xi}{\sqrt{1-\xi^2}} \sin \omega_j \sqrt{1-\xi^2} t_d \end{bmatrix},$$

$$(j = 1, 2), \text{ for } 0 \leqslant \xi < 1. \qquad (9)$$

The FSMC control loop is shown in Figure 3. The control loop starts when the control system observes the states x_{1j} and x_{2j} of the beam structure for the jth mode. Then it calculates σ_j by equation (7), $\dot{\sigma}_j$ by equation (8), and then $S_1 = \sigma_j$ and $S_2 = \mathrm{d}S_1/\mathrm{d}t$. S_1 and S_2 are fuzzified and assigned to certain fuzzy variables according to the control rules. A defuzzification process is followed. Finally, an established loop-up table is used to look up the control force.

4. SIMULATION RESULTS AND DISCUSSION

The Runge–Kutta method is used to solve equation (6) with a numerical tolerance of 10^{-3} for each step. The system parameters for simulation are shown in Table 2. Based on the results of numerical simulation, we compare the performance of various control laws, namely (1) passive control and the FSMC, and (2) the FSMC with phase shift compensation.

4.1. PASSIVE CONTROL AND FSMC

Without the actuation of servomechanism, i.e., $u(t) = 0$ in equation (2), the system has passive control only. Responses of the beam structure under passive control and without control are shown in Figure 4. In this case, the system will become unstable without control. When passive control is employed, the system's amplitude is slightly reduced but cannot be eliminated. This shows that passive control is not a very good control method.

TABLE 2

Parameters for simulation

Beam span	$L = 100$ ft
Mass per unit length of beam	$m = 0{\cdot}3$ lb
Bending stiffness of beam	$EI = 12 \times 10^{10}$ lb in^2
Control torque location	$a = 10$ ft
Stiffness of control spring	$k = 62{\cdot}5$ kips/in
Length of spring arm	$l = 3$ ft
Dynamic load	$P = 20$ kips
Velocity of dynamic load	$V = 60$ ft/s

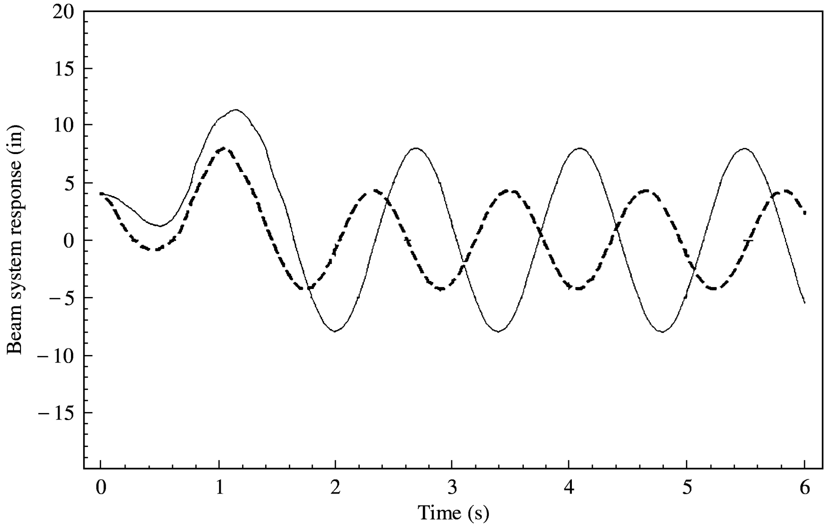

Figure 4. Response of beam structure: $t_d = 0{\cdot}1$ s; - - - - -, passive control; ——, without control.

Now, the FSMC is applied to actively control the same dynamic beam structure, but the time delay effect is not compensated. For the control system without time delay effect, the responses under passive control and the FSMC are shown in Figure 5(a). It shows that the FSMC will greatly reduce the system's response in comparison with passive control when the system has no time delay.

However, when the delay time is increased, the system will become unstable if the time delay is not compensated. The results are shown in Figure 5(b) in which the time delay is 0·1 s.

4.2. FSMC WITH PHASE SHIFT COMPENSATION

Now, to avoid the failure of the FSMC for the system with time delay effect, the FSMC with phase shift method is used to control the system and the time delay effect is compensated.

Here we compare the performance of the FSMC with phase shift compensation for different delay times. In Figure 6, the result shows that increase in delay time will enlarge the response amplitude, while the system remains stable. However, the FSMC with phase shift compensation can achieve good control for the system with time delay effect.

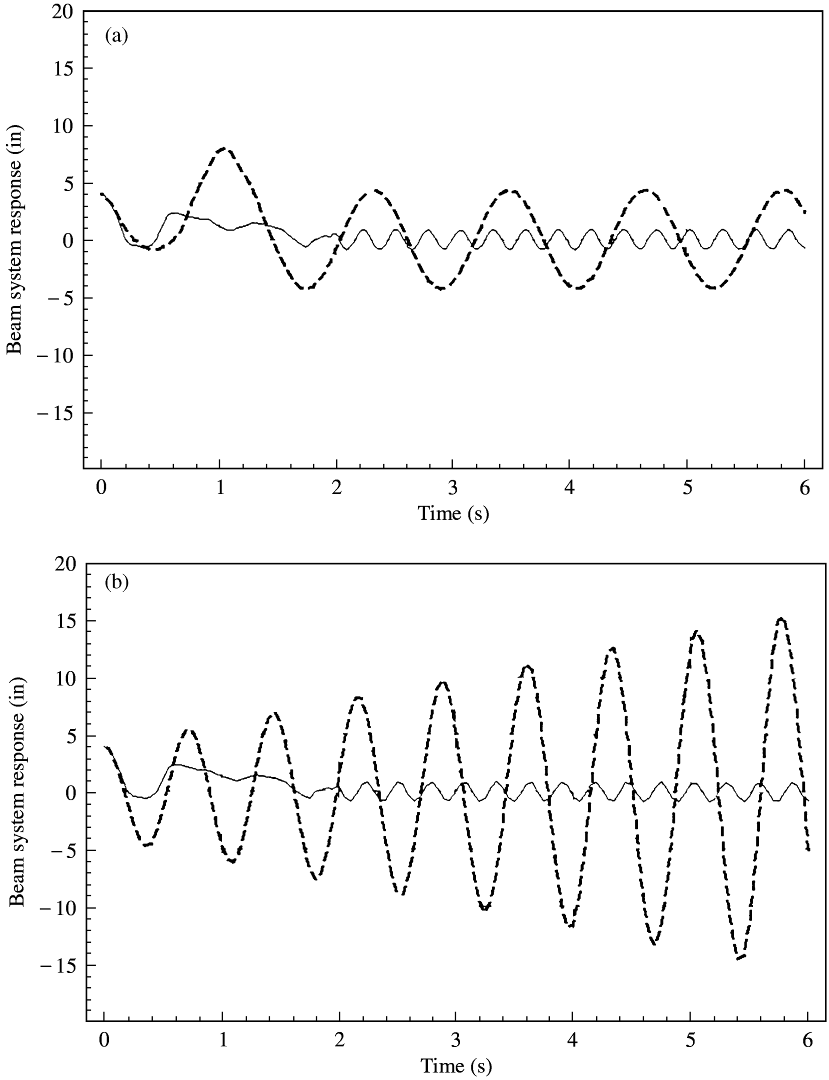

Figure 5. Response of beam structure: (a) $t_d = 0$ s; -----, passive control; ——, FSMC; (b) $t_d = 0{\cdot}1$ s; ——, FSMC; -----, FSMC with phase shift compensation.

It is noted that increase in delay time will reduce the system's performance. Obviously, the tolerance for delay time is another important characteristic of the FSMC method.

Furthermore, in order to estimate how much the FSMC can do and keep the system in control, the maximum and averaged bending displacement reduction rates, R_{max} and R_{ave}, are chosen for this purpose. They are defined as follows:

$$R_{max} = \frac{X_{max\text{-}uncontrol} - X_{max\text{-}control}}{X_{max\text{-}uncontrol}}, \tag{10}$$

$$R_{ave} = \frac{X_{ave\text{-}uncontrol} - X_{ave\text{-}control}}{X_{ave\text{-}uncontrol}}, \tag{11}$$

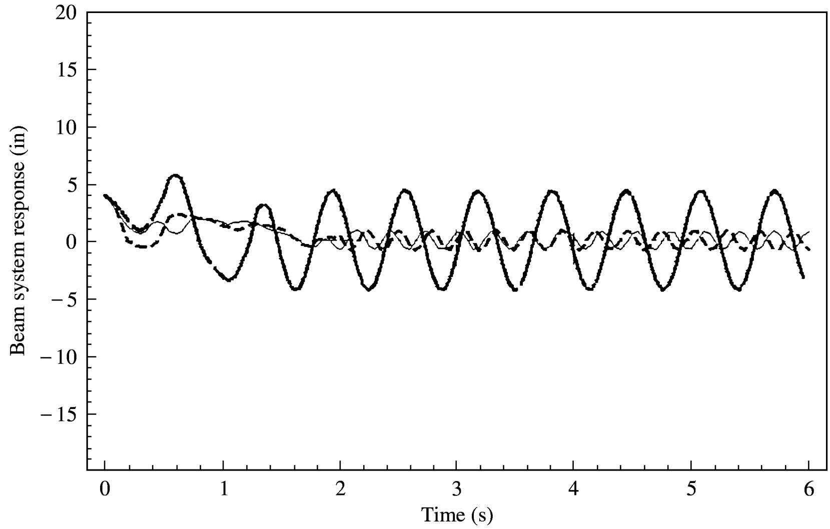

Figure 6. Time delay effect on dynamic beam system by FSMC with phase shift compensation: -----, $t_d = 0{\cdot}01$ s; ——, $t_d = 0{\cdot}01$; ▬▬, $t_d = 0{\cdot}2$.

where $X_{max\text{-}uncontrol}$ and $X_{ave\text{-}uncontrol}$ represent the system's maximum and averaged responses without control respectively.

During the loading session, the different reduction rates against time delay are shown in Figure 7(a) and 7(b). It is noted that increase in delay time will decrease displacement reduction rates and the system's performance becomes worse. For both the maximum and average reduction rates, the FSMC with phase shift compensation achieves better reduction rates in comparison with the FSMC without phase shift compensation. It shows that the FSMC with phase shift compensation could effectively reduce the displacements of beam structure with time delay effect. The results also show that the FSMC with phase shift compensation is capable of controlling the system with time delay up to about 0·2 s.

In summary, from the results shown above, it is shown that the FSMC together with phase shift compensation can enhance the system's robustness against the time delay effect.

5. CONCLUSION

The objective of this research is to present a feasible methodology to control a beam system with time delay effect. For this purpose, a simplified state-space model of a dynamic beam structure is chosen as the control plant. Usually, when the system has time delay effect, an inappropriate control force via typical control laws will result in poor control.

Here, we use the fuzzy sliding mode control to deal with the control system with time delay effect. The FSMC is easy to design and needs little online calculations. In comparison with a passive control system, the FSMC has better performance especially for systems with time delay effect. The results also show that increase in the system's delay time will reduce the performance of the FSMC, and the system will become unstable when the time delay is large. To achieve a better control performance, the phase shift method is applied to compensate for the system's phase lag caused by the time delay effect. The result shows that the FSMC with phase shift compensation will effectively improve the system's tolerance for dynamic stability.

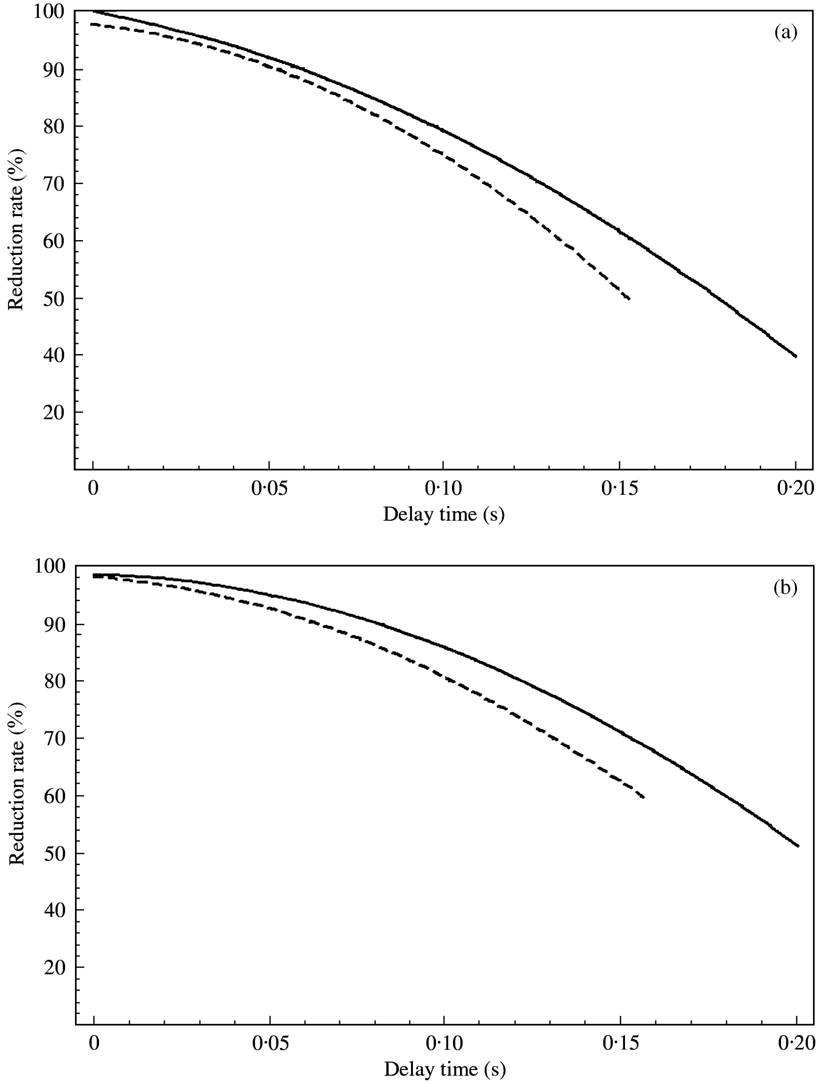

Figure 7. (a) Maximum beam bending displacement reduction rates versus delay time; (b) Average beam bending displacement reduction rates versus delay time: ——, FSMC with phase shift compensation; - - - - -, FSMC.

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APPENDIX A: NOMENCLATURE

$\mathbf{A}$	system matrix
$\mathbf{A}_1$, $\mathbf{A}_2$	system matrix for the first and the second modes respectively
a	distance between servohinge and end of beam
$\mathbf{B}$	control coefficient matrix
$\mathbf{B}_1$, $\mathbf{B}_2$	control coefficient matrix of the first and second modes respectively
b_1, b_2	parameters for beam equation of the first and second modes respectively
$\mathbf{C}$	output coefficient matrix
$\mathbf{C}_1$, $\mathbf{C}_2$	output coefficient matrix of the first and second modes respectively
$\mathbf{C}_j$	coefficient matrix of hyperplane ($j = 1, 2$)
c_{1j}, c_{2j}	coefficient scalar of hyperplane for the jth mode ($j = 1, 2$)
$\mathbf{D}$	disturbance coefficient matrix
$\mathbf{D}_1$, $\mathbf{D}_2$	disturbance coefficient matrix of the first and second modes respectively
$\mathbf{E}$	external disturbance
$\mathbf{E}_1$, $\mathbf{E}_2$	external disturbance of the first and second modes respectively
e_{1j}, e_{2j}	the error and error rate function of the state X_j
EI	stiffness of beam
L	span length of beam
l	arm length of servomechanism
$M_o(t)$	control torque produced by actuator
M_j	the set of hyperplanes ($j = 1, 2$)
m	mass per unit length of beam
R_{max}	maximum bending displacement reduction rates
R_{ave}	average bending displacement reduction rates
S	stiffness of spring
S_1	sliding function for the jth mode
S_2	derivative of S_1
T_j	phase shift matrix for the jth mode ($j = 1, 2$)
$\mathbf{U}$	control force of system
U_j	control force for the jth mode
$u(t)$	displacement of actuator
$\mathbf{V}$	velocity of moving load
$\mathbf{X}$	state vector of system
$\mathbf{X}_j$	state vector for the jth mode ($j = 1, 2$)
$X_{ave\text{-}control}$	average displacement under FSMC control
$X_{ave\text{-}uncontrol}$	average displacement without control
$X_{max\text{-}control}$	minimum displacement under FSMC control
$X_{max\text{-}uncontrol}$	minimum displacement without control
X_{jd}	state value of time delay system for the jth mode ($j = 1, 2$)
x	distance to left end of beam

x_{1j}	displacement of state variable for the jth mode ($j = 1, 2$)
x_{2j}	velocity of state variable for the jth mode ($j = 1, 2$)
$\mathbf{Y}_j(t)$	beam displacement for the jth mode ($j = 1, 2$)
$\dot{\mathbf{Y}}_j(t)$	derivative of $\mathbf{Y}_j(t)$ for the jth mode ($j = 1, 2$)
$y(x, t)$	beam displacement

Greek letters

ζ	damping ratio
δ	Dirac delta function
δ'	derivative of δ
σ_j	the sliding function of the jth mode ($j = 1, 2$)
$\dot{\sigma}_j$	derivative of σ_j
ω	angular frequency, $\omega = (\pi V/L)$
ω_j	angular frequency for the jth mode ($j = 1, 2$)